

New probes of anomalous $WW\gamma$ couplings at future e^+e^- Linacs ¹

K.J. Abraham², J. Kalinowski³, & P. Ściepko
Institute of Theoretical Physics
ul. Hoża 69
00 681 Warsaw
POLAND

Abstract

We investigate the sensitivity of single photon plus missing energy cross-sections at future e^+e^- linacs to non-standard $WW\gamma$ couplings. We show that even with conservative estimates of systematic errors there is still considerable sensitivity to anomalous couplings. Analytic expressions for helicity amplitudes are presented.

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One of the primary physics goals of the next linear collider will be a detailed investigation of the non-abelian sector of the standard model, in particular to improve LEP bounds on non-Yang-Mills like triple gauge boson vertices. Most studies uptill now have focussed on the reaction $e^+e^- \rightarrow W^+W^-$ [1], which despite being a sensitive probe of non-standard $WW\gamma$ and WWZ couplings suffers from the drawback that there is no obvious way to disentangle the effects of $WW\gamma$ and WWZ form-factors. Hence it would be desirable to investigate other channels where such $\gamma-Z$ interference effects are not present.

In this letter we suggest that the process $e^+e^- \rightarrow \bar{\nu}\nu\gamma$ may be used to study the precise structure of the $WW\gamma$ vertex without complications from interference effects described above. Such final states have been studied at PETRA [2] and at LEP [3] as a means of determining the number of light neutrini. At these energies though, not much sensitivity to anomalous $WW\gamma$ form factors should be expected, as s-channel processes mediated by virtual Z exchange also play an important role. However, at Next Linear Collider (NLC) energies ($\sqrt{s} = 500\text{GeV}$), s channel contributions become less important and the bulk of the cross-section comes from t channel W exchange. As we will show, it is possible to choose cuts which enhance the sensitivity of the observed cross-sections and differential distributions to the $WW\gamma$ form-factor.

How does one parameterise possible deviations from the standard model non abelian vertices ? As was shown by Hagiwara et.al [4], there are seven possible Lorentz and $U(1)_{em}$ invariant triple gauge boson form-factors. If we require C and P invariance then only two, traditionally denoted by κ and λ , remain. Neglecting the form factors violating C and P is reasonable for our purposes as in the absence of beam polarisation there is no way to detect CP violating asymmetries if the only particle detected is a photon of unknown polarisation. We assume that physics at a scale much higher than those directly accesible by experiment is responsible for these form factors. With this assumption, we may set κ and λ to be constant. In the standard model we have $\kappa = 1$ and $\lambda = 0$. Deviations from the standard model are then parameterised by $\delta\kappa = \kappa - 1$ and λ . The modified Feynman rules for the $WW\gamma$ vertex may be obtained from [5], all other Feynman rules are standard ones.

Due to the complexity of the Feynman rules for the non-standard couplings it turns out to be convenient to calculate the matrix-element using

the helicity amplitude formalism [6, 7]. The results are presented in the Appendix. Note that the helicity amplitudes for non-standard couplings have not been presented elsewhere. The helicity amplitudes for the standard model agree with those in Appendix A of [3]. Note however, that some of the analytical expressions in [3] are incorrect, in particular not all the terms in Eq. 3 have the same dimension. As a check we have independently evaluated the helicity amplitudes using the formalisms of [8] and [7] and find excellent numerical agreement for various values of κ and λ . As a further check we have verified the results for the differential distributions at the Z resonance presented in [3].

We are now in a position to calculate $\sigma(e^+e^- \rightarrow \bar{\nu}\nu\gamma)$ at NLC energies with non-standard $WW\gamma$ form factors. Since we assume the electron to be massless we need to impose a minimum angle cut on the direction of the outgoing photon to avoid colinear singularities as well as a minimum energy cut to avoid IR problems. Setting $\theta_{min} = 20^\circ$ and $E_{min} = 10\text{GeV}$ we find for the Standard Model a cross-section of $\sim 1.6\text{ pb}$, which at projected NLC luminosities ($\sim 10\text{ fb}^{-1}$) represents a sizable number of events. However, with these cuts alone the sensitivity to non-standard couplings is rather small. This is hardly surprising; the bulk of the cross-section comes from initial state soft photon bremsstrahlung and is thus only weakly dependent on the non-abelian couplings. Further cuts are required in order to enhance the relative importance of the $WW\gamma$ couplings. As the anomalous form factors are associated with higher dimensional operators containing derivative interactions, it is clear that only the more energetic photons will be sensitive to the additional derivatives. Therefore we require that the minimum energy of the photon be 80 GeV. In order to further improve the situation it is necessary to reduce the background from the Z exchange graphs. This can easily be achieved by means of a simple kinematical trick. In the limit that the Z width is negligible, the photon is essentially monochromatic, with an energy close to half the centre of mass energy. However, this corresponds to almost the edge of phase space, where the cross-section for the t channel graphs is close to zero. Hence we require that the energy of the photon be less than 180 GeV, which not only removes the background from Z exchange graphs but also does not reduce the signal from the WW exchange graphs too much.

With these cuts on the photon energy, the cross-section for the standard model is 0.21 pb, which still leads to an appreciable number of events at NLC luminosities. Cross-sections for non-standard values of $\delta\kappa$ and λ with the

cuts mentioned above are presented in Figs. 2 and 3. (We have deliberately varied $\delta\kappa$ and λ individually and not simultaneously, in order to keep the analysis simple.) As is clear from the curves, there is considerable sensitivity to deviations from the standard model. Precise discovery limits though, are dependent on what assumptions are made concerning systematic uncertainties, since statistical errors are probably quite small given the large numbers of events $\mathcal{O}(2000)$. Assuming there are no experimental systematic errors, the main source of theoretical systematic errors lies in unknown higher order corrections. Note that the higher order corrections discussed in [3] are those which are dominant on the Z pole, and are therefore not adequate for our purposes. It is reasonable to assume that the bulk of the corrections are real and virtual QED corrections which integrated over the bulk of phase space are probably quite small. However we are restricting ourselves to a limited region of the total phase space where radiative corrections may be sizable even though the total radiative corrections themselves are small. Therefore we make the conservative assumption that the systematic uncertainties due to radiative corrections is $\mathcal{O}(20\%)$.

With this assumption it is possible to put the following discovery bounds $-.6 < \lambda < .6$ and $-0.6 < \delta\kappa < 2.2$ using the cross-section with the cuts mentioned above as only sensitive variable. It is interesting to note that this compares favourably with the bounds obtained by McKellar and He [9] on the basis of the recent CLEO measurement of $b \rightarrow s\gamma$ [10]. Further refinement is possible if one considers differential distributions. This is illustrated in Fig. 3 where we have plotted the differential distribution with respect to photon energy for the standard model and for two values of $\delta\kappa$. Although the cross-sections are almost the same the differential distributions are different. Similar effect is observed for λ . It would not be wise though, to derive further discovery bounds on the basis of differential distributions without a detailed consideration of detector acceptances and higher order radiative corrections.

To conclude, we have demonstrated that $\sigma(e^+e^- \rightarrow \bar{\nu}\nu\gamma)$ at projected NLC energies and luminosities is sensitive to anomalous $WW\gamma$ couplings. Making conservative estimates for systematic errors involved it is possible to constrain λ and κ to lie within the regions $-.6 < \lambda < .6$ and $-0.6 < \delta\kappa < 2.2$. These bounds can probably be improved through knowledge of currently unknown radiative corrections. We have also for the first time presented all the relevant helicity amplitudes in a simple and usable form.

Upon completion of this work we became aware of a paper by Couture

and Godfrey OCIP/C 94-4, UQAM-PHE-94-04, addressing similar issues but containing only numerical and no analytical results. The discovery limits they derive are much more stringent than ours due to more optimistic assumptions about the size of theoretical systematic errors.

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Appendix

We consider the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ with the following kinematics (all momenta are physical, *i.e.* with positive energy)

$$e^+(p_1) + e^-(p_2) \rightarrow \nu(p_3) + \bar{\nu}(p_4) + \gamma(p_5) \quad (1)$$

The results for the helicity amplitudes can be conveniently expressed in terms of inner spinor products defined as follows

$$\langle ij \rangle \equiv \langle p_i - | p_j + \rangle = \sqrt{p_i^+ p_j^-} \exp[-i(\omega_i - \omega_j)/2] - \sqrt{p_i^- p_j^+} \exp[i(\omega_i - \omega_j)/2], \quad (2)$$

where $|p\pm\rangle$ denotes the fermion with momentum p and helicity \pm , $p_i^\pm = p_i^0 \pm p_i^3$ and $\exp(i\omega_i) = (p_i^1 + ip_i^2)/p_i^T$. The inner spinor product has the following properties [7]

$$\begin{aligned} \langle p_i - | \gamma^\mu | p_j - \rangle &= \langle p_j + | \gamma^\mu | p_i + \rangle \\ \langle p_i + | \gamma^\mu | p_j + \rangle &= \langle p_k + | \gamma_\mu | p_l + \rangle = 2 \langle p_k + | p_i - \rangle \langle p_j - | p_l + \rangle \\ \langle p_i + | p_j - \rangle &= \langle p_j + | p_i - \rangle^* \\ | \langle p_i - | p_j + \rangle |^2 &= 2 p_i \cdot p_j \end{aligned}$$

In the calculations it is convenient to take the polarization vector of the photon with momentum p and helicity \pm as follows

$$\epsilon_\pm^\mu(p, k) = \pm \frac{\langle p \pm | \gamma^\mu | k \mp \rangle}{\sqrt{2} \langle k \mp | p \pm \rangle} \quad (3)$$

with

$$\gamma_\mu \epsilon_\pm^\mu(p, k) = \pm \frac{|p \mp \rangle \langle k \mp | + |k \pm \rangle \langle p \pm |}{\langle k \mp | p \pm \rangle} \quad (4)$$

where k is the reference momentum when judiciously chosen may simplify the calculations significantly. We find that choosing p_2 (p_1) for photon helicity $+$ ($-$) as a reference momentum (with the exception for the diagrams with Z boson exchanges for positive helicity of the electron, first two lines below) one obtains particularly simple analytical expressions for the helicity amplitudes. We find for the Z diagrams:

$$\begin{aligned}
A_Z^{++} &= f_Z(g_v - g_a) \frac{\langle 23 \rangle^2 \langle 34 \rangle^*}{Z \langle 25 \rangle \langle 15 \rangle} \\
A_Z^{+-} &= f_Z(g_v - g_a) \frac{\langle 14 \rangle^{*2} \langle 34 \rangle}{Z \langle 15 \rangle^* \langle 25 \rangle^*} \\
A_Z^{-+} &= -f_Z(g_v + g_a) \frac{\langle 13 \rangle^2 \langle 34 \rangle^*}{Z \langle 15 \rangle \langle 25 \rangle} \\
A_Z^{--} &= -f_Z(g_v + g_a) \frac{\langle 24 \rangle^{*2} \langle 34 \rangle}{Z \langle 15 \rangle^* \langle 25 \rangle^*}
\end{aligned} \tag{5}$$

for W diagrams

$$\begin{aligned}
A_W^{-+} &= -f_W \frac{\langle 13 \rangle^2 \langle 34 \rangle^*}{W_{14} \langle 15 \rangle \langle 25 \rangle} \\
A_W^{--} &= -f_W \frac{\langle 24 \rangle^{*2} \langle 34 \rangle}{W_{23} \langle 15 \rangle^* \langle 25 \rangle^*}
\end{aligned} \tag{6}$$

and for WW diagram

$$A_{WW}^{-+} = A_W^{-+} \frac{\langle 45 \rangle^* \langle 23 \rangle^* \langle 25 \rangle}{W_{23} \langle 34 \rangle^*} \tag{7}$$

$$A_{WW}^{--} = -A_W^{--} \frac{\langle 35 \rangle \langle 14 \rangle \langle 15 \rangle^*}{W_{14} \langle 34 \rangle} \tag{8}$$

in the standard model, where the superscripts denote the helicity of the electron and photon, respectively, $g_v = -1/2 + 2 \sin^2 \theta_W$, $g_a = -1/2$ and

$$\begin{aligned}
f_Z &= \frac{ieg^2}{\sqrt{2} \cos^2 \theta_W} \\
f_W &= -ieg^2 \sqrt{2} \\
Z &= (p_3 + p_4)^2 - M_Z^2 - iM_Z \Gamma_Z \\
W_{14} &= (p_1 - p_4)^2 - M_W^2 - iM_W \Gamma_W \\
W_{23} &= (p_2 - p_3)^2 - M_W^2 - iM_W \Gamma_W
\end{aligned} \tag{9}$$

For the additional contributions from WW diagram with anomalous $\delta\kappa$ and λ couplings we get

$$A_{\kappa}^{-+} = +\delta\kappa f_W \frac{\langle 25 \rangle^* \langle 45 \rangle^* \langle 13 \rangle}{2W_{14}W_{23}} \quad (10)$$

$$A_{\kappa}^{--} = -\delta\kappa f_W \frac{\langle 24 \rangle^* \langle 35 \rangle \langle 15 \rangle}{2W_{14}W_{23}} \quad (11)$$

$$A_{\lambda}^{-+} = \lambda f_W \frac{\langle 45 \rangle^* \langle 14 \rangle \langle 24 \rangle^* \langle 25 \rangle^* \langle 23 \rangle}{2M_W^2 W_{14}W_{23}} \quad (12)$$

$$A_{\lambda}^{--} = -\lambda f_W \frac{\langle 23 \rangle^* \langle 35 \rangle \langle 13 \rangle \langle 14 \rangle^* \langle 15 \rangle}{2M_W^2 W_{14}W_{23}} \quad (13)$$

$$(14)$$

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Figure Captions

Fig.1. Cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ as a function of $\delta\kappa$ for $\lambda = 0$

Fig.2 Cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ as a function of λ for $\delta\kappa = 0$

Fig.3. Energy spectrum of the photon at $\sqrt{s} = 500$ GeV for the standard model and for $\delta\kappa = -0.6$ and 2 .

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